A course on peer methods for the solution of nonstiff and stiff ODEs

We consider the numerical solution of

\[ y' = f(t, y), \quad y(t_0) = y_0. \]

We discuss the problem of stiffness and give an overview about classical numerical methods:

- Linear multistep methods: Adams methods for nonstiff and BDF for stiff problems.

Peer methods belong to the wider class of General linear methods (GLM) and have been studied intensively in the last few years. They can be interpreted as two-step multistage methods. Peer methods have some favorable properties:

- Their stage order is equal to their order. They do not suffer from order reduction for very stiff problems.
- The construction of parallel methods of high order is very easy.
- They allow dense output.

We will consider

- Explicit peer methods for nonstiff problems,
- Implicit peer methods for stiff problems,
- Exponential peer methods for special MOL problems.

In this course we will give an introduction to peer methods with focus on

- Order of consistency and convergence,
- Stability (Zero-stability, A- and L-stability),
- Implementation issues (Solution of the nonlinear systems in implicit methods, approximation of exponential matrices in exponential peer methods),
- Software (in Matlab and in Fortran), numerical tests and comparisons with standard methods.